THE Partnership | Improving Schools Together

Teaching multiplicative reasoning through problem solving

A teacher research project with Tower Hamlets Schools 2022

Led by Mike Askew

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Project Commissioned by

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Content

Introduction

A focus on multiplicative reasoning

Traditionally the initial focus in number work in primary schools is based around Additive Reasoning (AR) with the assumption that a sound base in addition and subtraction leads into multiplicative reasoning (MR), which is seen as more challenging than AR.

Evidence shows, however, that MR is not actually more difficult than AR - it is simply a different way of understanding a relationship between quantities. That, for instance, the relationship between two cups and six cups can be thought of additively (six cups is four cups more than two cups) or multiplicatively (six cups is three times as large as two cups) (For a detailed overview see (Nunes & Bryant, 2009a, 2009b) There is also evidence that MR can be productively worked alongside AR (Askew, 2018). For example, two key ideas underpinning MR that differ from AR are:

- Measures – if I can fill four cups from a bottle, then the bottle is four times as big as the cup, and each cup is a quarter of the bottle.
- Scaling – if a seedling is 1 cm tall and it grows to be four 4 cm tall, then it is four times as tall as it was.

Children can work with both these of ideas from Reception upwards, and in revisiting them in later years deepen their understanding of MR. These ideas also connect to understanding division, fractions, place value and measurement, and so different strands of the mathematics curriculum become woven together. There is also evidence that children with a good sense of MR not only have a better understanding of fractions, but also, later, have fewer difficulties when encountering algebra (Schmittau, 2005). Thus working with MR results in deep understanding and enhanced reasoning across the years.

A focus on teaching through problem solving

The approach taken to examine the teaching of MR in this research partnership was a modified form of lesson-study, wherein the teachers designed lessons based around a problem intended to engage the children in informally thinking about MR, and from the children's solutions to the problem the teacher could draw out key MR ideas.

After two days of PD exploring the key ideas behind MR, the participant teachers worked in four teaching team groups, each designing a lesson based around a key idea in MR. The design of the lessons engaged the teaching team in an iterative process of:

- Choosing a problem that they expected would engage the children in a key aspect of MR: the problem should provoke the children to produce a range of informal responses, not simply be the application of prior learning.
- Anticipating the sorts of responses and solution methods that the children might be likely to come up with.
- Considering whether the anticipated responses were likely to provide the raw material out of which the lead teacher could draw out the key MR ideas. If not, returning to looking at the problem and considering how it might be modified to produce more generative results from the children.

These lessons provided the basis of a modified lesson study approach which took the following form:

- One teacher in each teaching team volunteered to teach the problem solving lesson on behalf of the group - the lead teacher in the accounts that follow. The other teaching team members would, during the teaching of the lesson, closely observe the children's responses to the problem and support the lead teacher in selecting which solution methods would be shared with the whole class. They would not, however, intervene to support or direct the children's efforts in any way.
- The other participating teaching would observe the lesson, but be totally 'hands off' with respect to engaging with the children.

Each lesson was designed a common structure

Phase 1 - Posing the problem

The lesson to start with all the children together and the lead teacher talking through the problem. While the problem might eventually be presented on the board, time was to be spent talking about the context of the problem and making sure everyone had a sense of what exactly the problem was before any formal presentation of it

Phase 2 - Independent working

The children work, individually or in pairs, to find a solution to the problem. During this time the teaching team would intervene as little as possible in what the children were doing. They might ask the children to explain their thinking, but only to inform the choice of solution methods to be shared and discussed.

Phase 3 - Sharing thinking

The lead teacher to orchestrate the sharing of solutions that had been selected, drawing the children's attention to similarities and differences between these.

Phase 4 - Drawing together

The lead teacher, drawing on the different solution methods, to make explicit the key MR ideas that emerged from the problem and its solutions.

Each lesson-study meeting took place over half a day, one lesson in a morning, one in the afternoon.

The lessons took place at 4 schools, with the participants moving between schools over lunchtime.

The participants met at the school before the lesson, during which time the planning group recapped what the major problem of the lesson would be and key aspects of MR that were anticipated to be focused on.

After the lesson, all participants discussed the key things that they had noticed and thought were significant. The lead teacher shared their observations first, followed by comments from the teaching team. Finally the whole group engaged in the discussion.

In the final day, the teachers worked within and across the teams to reflect on the significance of the individual lessons and the overall learning. The rest of this report presents an account of each of the four lessons, followed by reflections on what was learned regarding learning, teaching and this as a form of professional development.

Part 1: The Lessons

Reception: Making Equal Groups

Context

- Host school: Blue Gate Fields Infant School
- 3 form entry Infant school in Shadwell, East London
- Reception class of 30 children
- Mixed attainment pairing for the lesson
- 29 children have EAL, Bengali predominantly home language
- 5 pupil premium - 5

Prior Learning

- Much learning missed due to Covid - reduced time in Nursery and a Covid outbreak in the class leading to online learning and homework packs
- The Maths Focus has been subitising, ordering, recognising, composting and de-composing numbers 1-5
- The children have had some experience of talk partners but were finding it difficult.

The Problem

Jack needs help to work out how many magic beans he can plant in his field.

When his field is full of beans, he needs help to put them into groups of five. Can you help him?

Key Questions

- Were the children able to explain their thinking?
- Could they work out how many more or fewer beans they need to fill the field and use that vocabulary?
- Did they see any connection between the array for the field being 4 by 5 and the number of groups of 5 they could make?

Resources

Children were given 4 by 5 arrays to represent the field and a bag of beans - these contained fewer or more than 20 beans.

The teacher had prepared a letter from Jack, linking the lesson to a story book being read with the children that week.

When it came to working out how many groups of five could be made, children were given large sheets of paper and marker pens to record with.

The Lesson

Phase 1 - Posing the problem

The lesson started, and continued throughout, with all the children in pairs on the carpet. The lead teacher set up that a letter had come from Jack, and one of the children read it out to the class. The lead teacher clarified that they were first to figure out if each bag had enough beans in it to fill the field, and, if not, that they needed to ask for how many more or fewer beans they needed to fill the field. They were then to work out how many envelopes they needed to store their beans if there were to be five beans in each envelope.

Phase 2 - Independent working

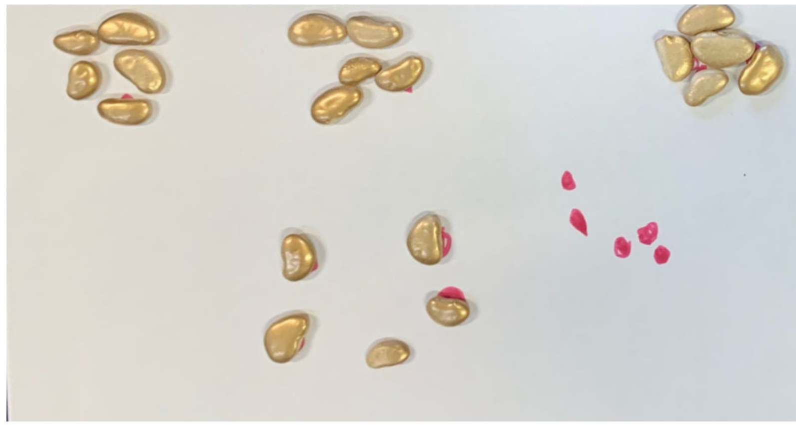
Children worked with their partner to find out whether the bag of beans they had been given filled the field and, if not, how many more or fewer beans they needed. During this part of the problem the children were expected to use their skills in counting up or down to 20.

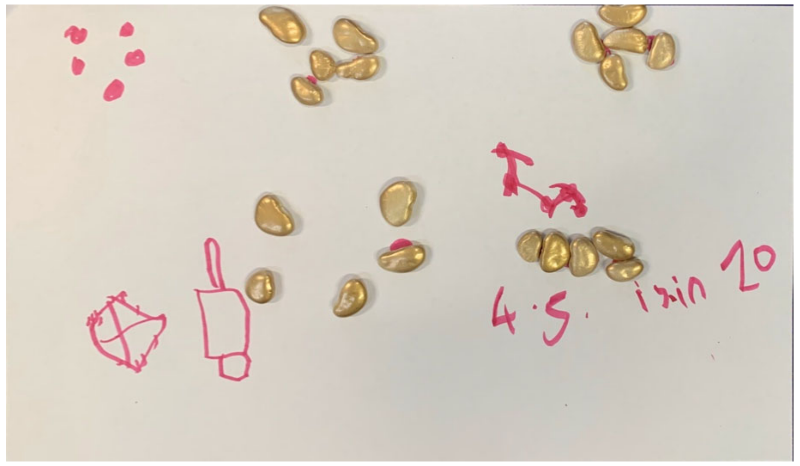
To get the required number of beans they had to tell the Lead Teacher or TA whether they needed more or fewer beans, how many more or fewer and, if they could, explain their request. They were given the number of beans they requested and so could then self-check if that was the correct number needed by adding or removing the beans to or from the array.

The sort of explanations they gave included:

- I need nine more, no nineteen more
- We don't need those two.
- We knew it was 5 because we subtilised', (saying this while pointing to a row of five empty squares on the array).

The children were then reminded of the second part of problem - that they needed to know how many packets they needed to pack all 20 beans into packs of five. The big idea in this part of the problem was to shift to recognising a group of five as a single composite unit as opposed to five single ones.





Once pairs had figured out how many packets they would need they were given paper and pens to record what they had done. Ways this did this included:

- Writing 5, 5, 5, 5
- Saying and recording 'Four 5s is in 20"
- Placing the beans in groups of five on the paper and drawing around them.

Phase 3 - Sharing the thinking

Children were invited to the front to share their thinking with the rest of the class.

Phase 4 - Drawing it together

The lead teacher recapped what the problems were and some of the strategies used.

Reflections on the lesson

Expectations

Do not set limits on expectations of children's capability to solve the problem.

Children were able to work with larger numbers even without prior school teaching experience.

When hooked into a context and problem even Reception children can be focussed on a task for an extended amount of time.

Preparation

Good preparation in terms of thinking about the numbers involved and the resources provided were necessary to ensure children's success.

Choice of problem: Problems need to be relevant (realisable, in the sense of being able to model) to the children.

Teaching

The session needed to flow naturally rather than imposing strict timings.

Teaching came after solutions and methods have been found.

A problem might run across more than one lesson to allow children adequate time to grapple with it.

Do not interrupt children working with unnecessary teacher talk.

Reflections on what we learnt

Expectations

Children are more capable than they are sometimes given credit for. For example, we had not expected reception children to reason along the lines of 'four fives is in 20'

Even though young, other children can ask and answer questions around the process and solution

Choice of problem

Problems set must have real meaning for the children to hook them in. Real meaning can be imaginative and fantasy, it is not the same as real-life.

Teaching

Allow children to share how they have solved the problem in their own words. Only ask questions for clarification. Do not talk for them.

Value of misconceptions: Misconceptions can be addressed as a class, by all the children as strategies are being shared. Celebrate their identification.

Year 4: Thinking down and up

Context

- Host school: Olga Primary School
- Two/three form entry school in East London
- Year 4 class
- As the class has a large cohort of SEN children we made the decision to have those children do a separate activity in another room. This, together with absences due to covid left 20 children in the lesson.
- 12/20 children EAL
- Age related expectations: 3 – greater depth, 11 working at, 6 working towards
- The children were in carefully chosen mixed ability pairings (they had always worked in mixed ability pairings so this was not different for this lesson).
- During the lesson the teacher and the TA supported the children. The TA was briefed to not help the children but to encourage them to put their ideas on the paper and to get them to explain their thinking to him.
- The teachers who co-planned the lesson observed the children and helped to select the work that would be good shared in the whole class discussion but did not interact with the children.

Prior learning

- The children had just begun a unit on fractions and had had three lessons on equivalence which included making equivalent fractions using playdoh and working with fraction walls.
- During coronavirus, the fractions unit was taught in school in both Year 2 and Year 3 so the children did have a good base of learning.

Key questions

- What strategies did children use in solving a fraction problem in a realistic context?
- What did these reveal about the children's understanding of fractions?
- Could the children solve a problem collaboratively?

The Problem

I have a bag of Monster Munch. I eat 1/4 and I have 12 Monster Munch left. How many Monster Munch were in the bag to begin with?

Resources

The children were given A3 paper and marker pens to record their representations. They were also given coloured cubes (in two colours), double-sided counters in Monster Munch bags, strips of paper.

The Lesson

Phase 1- Posing the problem:

The children in the class already knew that the teacher loves crisps and about the teacher's Crispmas advent calendar. The children were asked to think back to 17th December when it was Christmas dinner day and the teacher had also eaten a second Christmas dinner at a party that evening. The premise was then set up that the teacher was very full but needed to taste the crisps to review them for her blog.

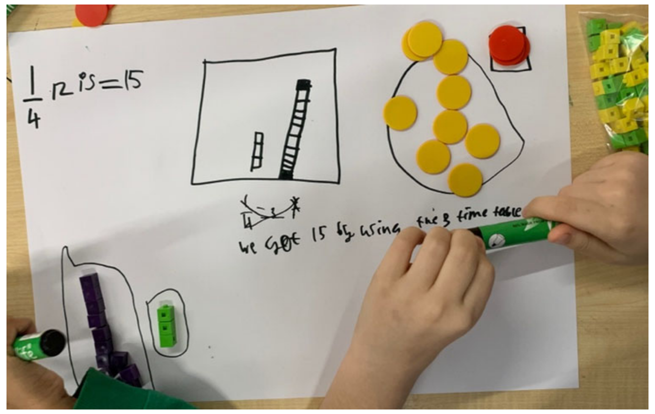
Phase 2- Pupils working independently

The children were given 15 minutes to try to solve the problem. The adults did not support the children in any way, while the lead teacher encouraged them to show their working in whatever way they wanted. The I had teacher and TA asked the children to explain their thinking but did not intervene by suggesting ways of working. All of the children were able to find a solution. Discussions between the teachers who had designed the lesson focused on selecting interesting methods

The majority of the children appeared to have found the correct answer.

However at least half of these answers were arrived at by simply adding the denominator of the fraction in the problem (¼) to the other number in the problem (12) and so were correct only by happenstance. We only realised that this was the dominant strategy when listening to what pairs were saying and in conversation with the children when it became apparent that they had no idea why they were adding 4.

We had expected that children would reason that 3 being ¼ of 12 would lead them to adding 3 to 12 and arriving at 15 at the answer. In the event, only one pairing added 3 to get 15.

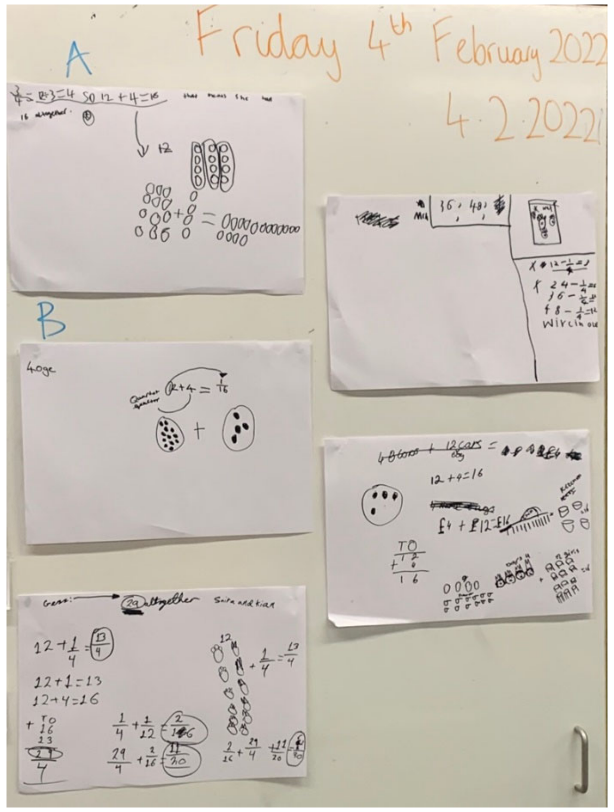


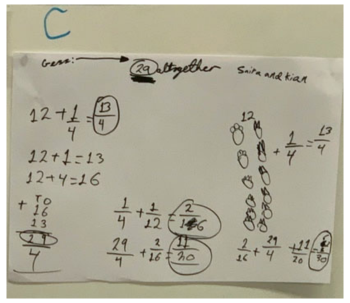
One pair had an answer of 29, reasoning that 12 had to be added to the numerator of 1, giving 13, then also adding 12 to the numerator of 4, 16, then adding the 13 and 16. It did not seem that this pair considered that this answer might be unfeasibly large.

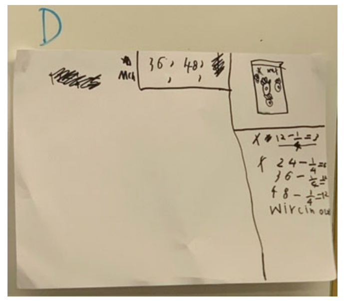
A final pair multiplied the remaining 12 by the denominator and were then unsure if the answer was 48, or whether the answer was 36 (¾ of the 48).

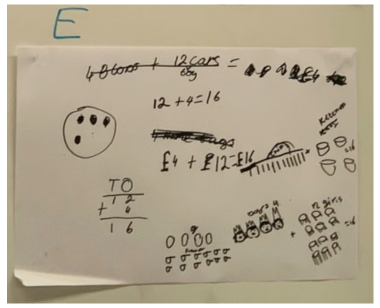
Phase 3 – Sharing and thinking

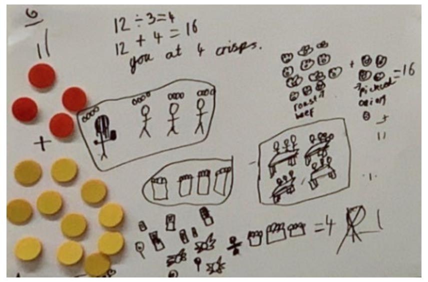
Initially all the solutions were stuck on the board with the lead teacher briefly explaining what the examples showed. The children were asked to group the methods that showed the same solutions and as this discussion unfolded the solutions were moved and re-grouped on the board, with similar solution methods put together and these groups lettered A – G.











The class then began to make sense of what each method showed. The discussion started with the incorrect answers and ended with the correct answer.

The teaching approach was not to choose the child who had produced the work to explain what they had done, but to ask other children to try and work out how an answer had been reached. The lead teacher picked up on misconceptions and guided the children to clarify what they meant. This meant slowing the pace of discussion to make sure, for example, that what being referred to by the four if a child said ‘they added four’ - in other words what the numbers being talked about were actually referring to in the problem (to some children the referents seemed still to be not clear even at the end of the discussion).

Phase 4 – drawing together

When it became clear that the children were not going to lead themselves to a full understanding of the 12 being ¾ the teacher put a bar model on the board to represent this.

¼ eaten amount	12 left		
?	4	4	4

Reflections on the lesson

Anticipating solution methods

In designing the lesson, we had spent a lot of time thinking of potential solutions the children would produce. However, we did not anticipate that they would simply add the denominator and by chance they would stumble upon the correct answer despite this incorrect method. Had we, for example, used ½ eaten and 10 left then the answer 12 obtained using this reasoning would have been likely to clearly show that this method was incorrect.

It was notable that the children were very focussed on their own work and that they didn’t copy from each other. They also were very good at working together and discussing what they thought.

Teaching approach

The lead teacher was nervous to annotate the children’s work and to ‘teach’ too much. If this lesson was taught again then more teacher intervention might have ensured that all children understood that the 12 was ¾ rather than the whole.

When the lead teacher put the bar model on the board, she did not write 16 as the total amount. This was a missed opportunity to make completely clear the maths behind why the answer was 16 not 15.

Choosing resources

The children did not really use the available resources – however we do wonder whether they were necessary given the small numbers.

A few children were preoccupied with the weight of the pack (perhaps from being given actual packaging), which led to them getting distracted from reasoning about fractions.

If I repeated the lesson, it would be useful to get the counters out to represent the solutions to disprove the answers.

Reflections on what we learned

Prior learning

The children brought far more prior learning to the problem than expected.

Pupil engagement

All of the children were able to produce a solution – we were concerned that they would be resistant to putting their ideas on paper but this proved to be an unfounded fear.

The children showed incredible stamina and focus.

The pairings were useful to help the children find solutions and work together.

Pupils’ explanations

The children were able to explain the other solutions but some did struggle to fully reason or justify their thoughts.

It was helpful to have the children explain others’ thoughts rather than their own as it meant they were really thinking about the solutions and how the other children had got to their answer.

Year 5: Exploring equivalence

Context

- Host school: Blue Gate Fields Junior School :3 form entry junior school in Shadwell, East London
- Year 5 class of 26 children
- All working at or above age related expectations – maths is streamed in school
- Random pairing within class
- Predominantly Bengali speaking children
- Pupil premium – 8, SEND - 2 children
- Taught by two teachers, one taking the lead role.

Prior Learning:

- First time exploring fractions in year 5
- Y4 fractions unit taught over home learning blogs during Covid and therefore disrupted. These children had revisited fractions when they returned to school after year 4 re-opened, focusing on what a fraction is, fraction of amounts and basic equivalent fractions
- The children had experience of collaborative work and were used to explaining their understanding.
- They had done lots of reasoning through prompts such as ‘prove it’ ‘convince your partner’ ‘convince your teacher’

Key questions:

- Were the children able to explain their thinking?
- Could they convince the teacher/or their pairs?
- Did they reach the conclusion that the fractions are the same and therefore that the girls and boys got the same amount of chocolate?

The Problem

I was at a birthday party for my niece and nephew, who are twins. It was a bowling party and afterwards they sat down to have some food. I was given the job of distributing some chocolate between the children. Initially I was going to share the bars equally but there were more girls than boys so I decided to give the girls more.

I had 15 bars and gave 9 bars to the 12 girls and 6 bars to the 8 boys.

My nephew told me this is unfair because the girls got more bars.

Was he correct? Was it fair or unfair?

Resources

Children had access to Cuisenaire Rods and strips of paper in varying lengths.

The Lesson

Phase 1 - Posing the problem

The lesson started with all the children on the carpet. The way it was posed was through a numberless scenario involving children sharing chocolate bars at a party. During the introduction of the problem, the children were asked questions to engage them and check their understanding. Partner talk was used and children’s ideas were shared on the carpet. They asked questions for clarification, such as

How many pieces did the chocolate bars have?

What chocolate bars were they?

Teacher 2 scribed the number of girls and boys and the numbers of bars given as a prompt for when children began to tackle it.

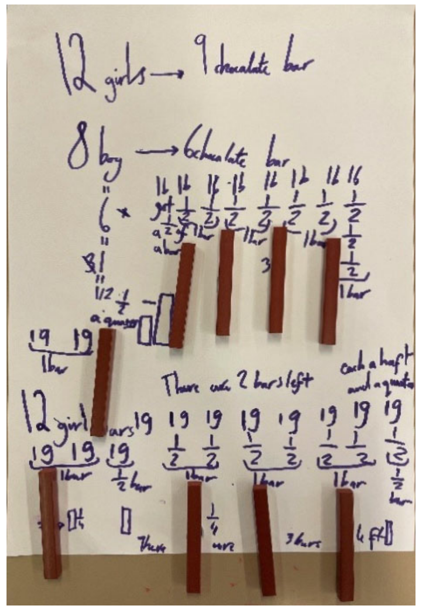
As the children were about to go off to work on the problem, one asked again how many squares there were in each chocolate bar. The lead teacher said that she was not going to tell them that.

Phase 2 - Independent working

The children went to their tables in pairs and began to work on the problem on A3 paper. There were many different outcomes:

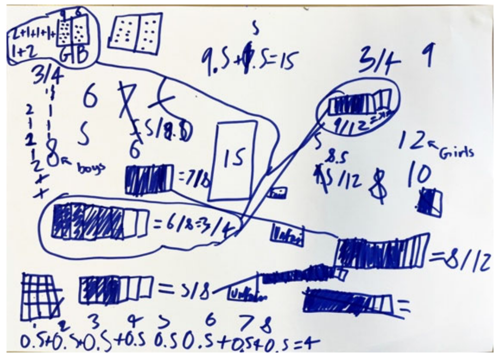
Lots of the children took Cuisenaire rods to represent the chocolate.

One pair were able to use the Cuisenaire rods to represent the problem and come to the conclusion that each child got half and a quarter.

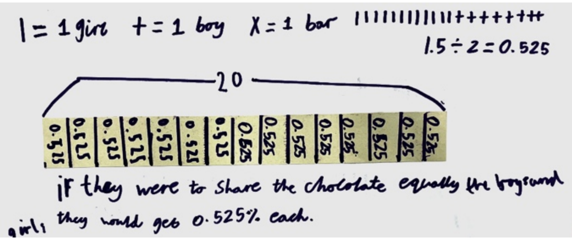


For many children, the Cuisenaire rods proved to be unhelpful with children becoming stuck when they couldn’t break the rods or could not find a way to use them to represent the problem. Part way through the lesson, the lead teacher advised against using the rods and all children were given strips of paper.

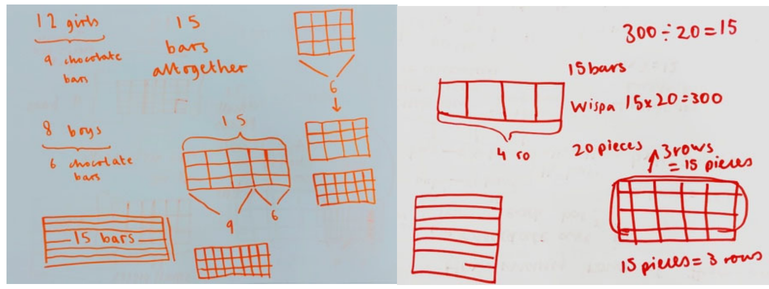
Some children tried to represent the bars as bar models. One child quickly drew two single bars that showed 8/12 and 6/9 but could not explain his understanding. He and his partner then tried lots of different ways to show/prove it.



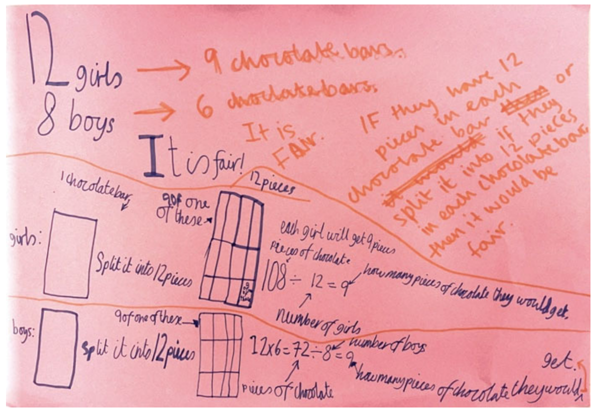
Another child drew a bar model showing 20 (total children) divided by 15 (bars of chocolate). He correctly reasoned that each child got half of 1.5 bars and that 1.5/2 = 0.5 and 0.25. This was recorded as 0.525.



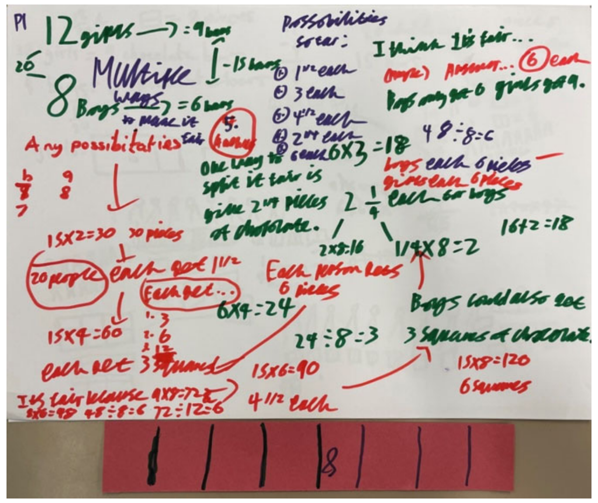
Two groups of girls attempted bar models that explored the different possibilities of pieces in a bar.



It became clear that the children were fixated on how many pieces of chocolate each bar had, which led some of them to explore different possibilities with multiples. For example, 1 child worked out that if each bar had 12 pieces, then it would be fair because 12 pieces x 9 bars = 108 / 12 children = 9 pieces per girl and 12 pieces x 6 bars = 72/8 = 9 pieces per boy.



Another child found multiple possibilities and realised that you could use any multiple of 4 as number of pieces to prove that the problem was fair.



Very few children manipulated the strips of paper. Those who did only chose to do so once encouraged by a teacher.

Phase 3 - Sharing the thinking

Examples were displayed in front of the class and the children were asked to share their thinking. The children as a whole thought that it was fair but not all were able to articulate their understanding in the time frame given during the lesson.

Child A - Shared that they worked out there were infinite possibilities because if the number of pieces of chocolate is a multiple of 4, then it can be shared fairly.

Child B - Explained that the answer of 0.525 was based in understanding that it was 0.5 and 0.25 added together.

Child C - explained that they shared 1 bar (represented as 1 rod) between two girls which meant they had half each. Then with the remaining 3 bars, had had to share them between 12. 12/3 = 4 so the girls got another quarter each. So that was 1 half and 1 quarter each, and it worked out the same for the boys.

Phase 4 - Drawing it together

The lead teacher used Child C's answer to guide the children to an understanding or 'answer' to the problem delivered – Was it fair? Modelling the paper strips as bars in front of all the children helped establish that the boys and girls each got ¾ of a chocolate bar.

Most children understood why it was fair but there was still a lot of questions around the number of pieces of chocolate in each bar!

Reflections on the lesson

Choice of problem

We wanted to choose a problem with a real-life context that the children could relate to and were invested in - this problem succeeded in doing that. We chose to introduce the problem as a numberless problem to generate enthusiasm and interest before the numbers were introduced. We chose chocolate bars because they are recognisable to children and bar models are used consistently in school

We chose a fraction problem because they had not yet learnt about fractions in year 5 and the problem provided a good opportunity for teaching through problem solving. It also provided an even playing field in terms of prior knowledge. Both of these came about in the enactment of the problem.

We wanted the focus of the lesson to be on showing their understanding rather than reaching an answer. The problem we chose allowed the freedom to represent solutions in different ways, however there was an 'answer' to guide the lesson. I.e. Yes, it was fair but why? While the problem did do this to a large extent, it seemed, however, that the desire of the children to know how many pieces there were in each bar of chocolate led to the focus on that, with some wanting to find a numerical answer to the problem rather than working with fractions to show equal quantities

Variety of representations and methods

The children produced a range of different responses to the problem, many of which we had not anticipated. The children spontaneously drew on different areas of maths and looked for patterns in numbers. E.g. relationship between the number and the 4 x tables.

Stamina

The children were invested throughout (and beyond) the lesson and demonstrated a real stamina and determination.

Resources

The decision to offer the use of Cuisenaire rods was a late addition to the plan and these proved not to be a helpful resource. They distracted the children and took up much of the problem-solving time because the children were fixated on using them. This led us to reflect on part of the role of the teacher in a teaching problem solving lesson is to recognise when learning is deviating for the intended trajectory and step in to move the learning forward - we like the idea of a productive path of learning. Strips of paper would have been a much clearer way for children to manipulate the bars of chocolate because they are tangible. If this problem was to be taught again, only offering strips of paper would be helpful.

Time

In ordinary circumstances, this lesson could have spilt over into following sessions. The teachers felt that they wanted to reach an end point because of the nature of the observation, which meant focussing on one outcome rather than exploring the variety of strategies used in the class. This lesson did, however, provide a really interesting insight into the children's understanding of fractions to build on in future lessons.

Setting up the problem

During the delivery of the problem one child asked how many pieces the chocolate bars had. The lead teacher said, "I'm not going to tell you that". This seemed to lead many children to believe this was an important piece of information and shaped the way they tackled the problem. On reflection, we would make it clear that the bars did not have pieces. This was addressed mid-lesson but by this point the majority had found a conclusion. If posing the problem again, it would be helpful to talk about the bars as not being split into separate pieces, but just being single 'slabs' of chocolate to discourage children from wanting to work with specific numbers, and then they might have been more inclined to use bar models. So while we anticipated that children would use variations of bar models to tackle the problem based on the idea of sharing the whole bar between the number of people in the event this did not happen as much as we had expected. Another possibility would be to physically fold a piece of paper or show children a bar with no pieces.

Reflections on what we learnt

Time

The idea of a 'lesson' puts constraints on the learning that can come out. Although we reached an answer to the problem delivered (that it was fair) , there was so much more to discuss and explore based on the children's outcomes during the lesson. When teaching through problem solving, you need time to fully immerse in the problem and the different directions that **could** be explored. Time to share and deepen through these and make links between the different strategies is needed. As a teacher, this means coming away and really looking at the children's responses before 'closing the lesson'. Even after the lesson, the teacher saw things that they hadn't noticed during the time frame.

Productive path

Not to be nervous to intervene in the children's solution path if it isn't looking productive. Equally, do not be nervous to move away from the direction you think, as the teacher, it should be going if it feels productive.

Carefully chosen numbers are important

The numbers that we chose meant that the problem was accessible for all children and, had they manipulated the strips of paper, they would have been able to fold them appropriately.

Can you prove it?

Even when they've reached an incorrect conclusion. Children 'disproving' their own misconception is a valuable way for them to learn.

Not underestimating children

Although we thought carefully about the responses and behaviours, we perhaps underestimated the children's stamina and the range of responses.

Focusing on numerical responses

Do we provide enough time and encouragement in teaching generally for children to explore finding solutions in different ways, particularly ones that do not rely on specific numbers? The children's eagerness to create equations suggested that they felt most comfortable working with specific numbers (rather than exploring relationships between quantities) and that maybe they need more time to explore mathematics problems with more freedom for how they represent solutions.

Year 6: Exploring volume

Context

- Host school: St Edmund's Primary School, a Catholic one-form entry school in East London.
- A mixed ability class of 25 children coming from a range of backgrounds with a high percentage of EAL learners.
- The school teaches a creative, topic-based curriculum that links across different areas of learning.
- The lesson was co-taught by two lead teachers
- The children were put into randomly selected mixed attainment pairs.
- The lesson was taught relatively soon after post lockdown-related home learning.
- The lesson took around 1 hour.

Prior learning

- The children had not been taught, in Y6, any units of work on volume, area or 3D shapes
- The children were used to working collaboratively

Key questions

- Did the children come to appreciate the importance of the different dimensions, length, width, and height of cuboids?
- Did they see a way to calculate the volume of a cuboid given its dimensions?
- Did they realise that cuboids with the same volume can have different dimensions?

The Problem

After spending time in YR marking cuboids out of cubes, we (T1 and T2) got interested in whether different cuboids can be made with the same number of cubes. We now want you to explore this.'

Resources

The children were given large sheets of paper and marker pens to record their representations. Each pair were given 16, 20, or 24 cubes.

The Lesson

Phase 1- Posing the problem:

The two lead teachers orally introduced the problem, drawing on the fact that one of the teachers usually has a reception class, and their interest had been piqued when building cuboids with reception. The pairs were asked to initially make one cuboid and record it in any way they wished.

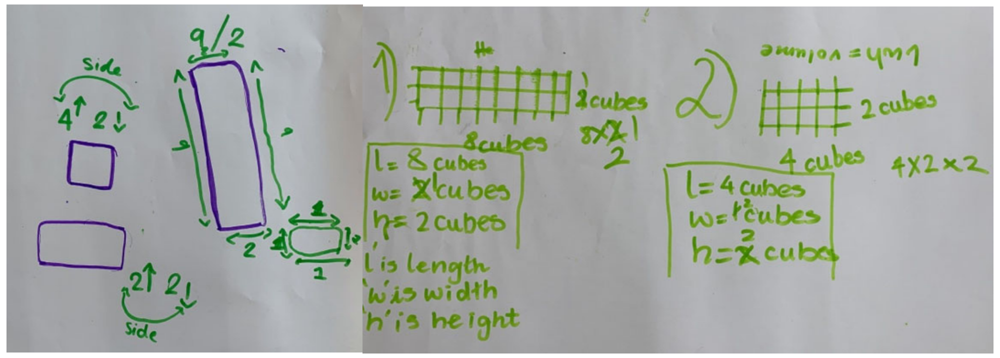
Phase 2- Pupils working independently

Each pair had 16, 20 or 24 cubes and were initially asked to make 1 cuboid. As the children made their first cuboids the lead teachers selected two pairs that had made cuboids with the same number of cubes but which had different dimensions. These were presented to the whole class and pupils asked to explain similarities and differences. Vocabulary that emerged in this discussion (length, width, height) was noted on the board. Once it was established that these two cuboids had different dimensions but the same number of cubes, the pairs were challenged to see if they could create three different cuboids using the total number of cubes they had been given and to find some way of recording these so that the differences could be seen.

As expected, a variety of different representations emerged, including

Representations of the faces of the cuboid by drawing around the physical cuboid.

Informal sketches of the 3D shapes.

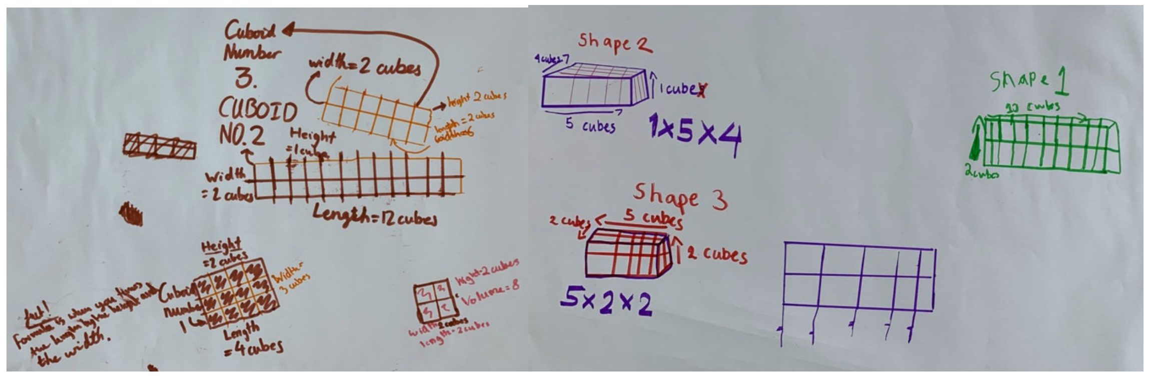


The majority of the representations initially did not include any numerical information.

To focus the children on whether or not their representations were helpful one, the class was stopped working and asked

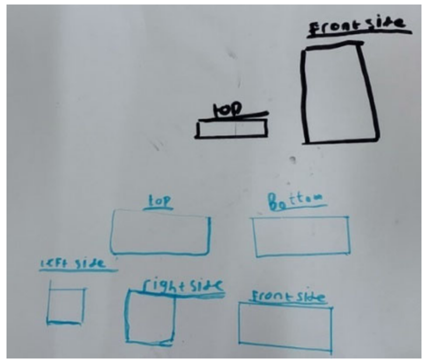
"Would someone else be able to build your cuboid based on your representation?"

Children went back to work and began to include more details and numbers in their representations.

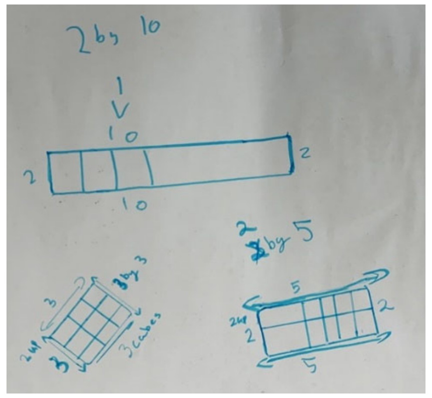


Phase 3 – Sharing and thinking

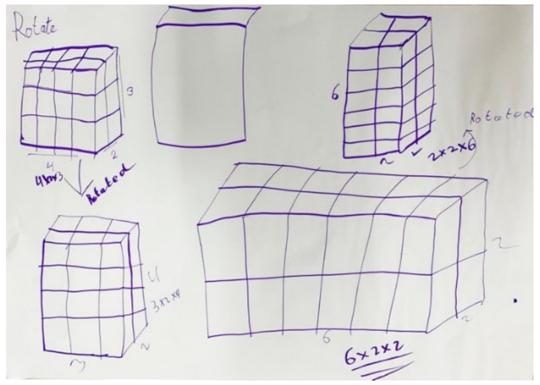
As the children developed their representations, the lead teachers chose 3 to share with the class:



A representation of each face of the cuboid, created by drawing around each face



An informal representation of the full cuboid, with some numbers showing dimensions



Another sketch of the cuboid accompanied by the notation 6 x 2 x 2

The children who had produced the representations were not asked to explain, rather the class was asked to speculate on what they thought each representation was shown and whether or not the cuboid could be re-created from the representation.

A key part of this discussion focused on the differences between length, breadth and width and what exactly these mean, particularly if a cuboid was rotated to change its orientation. Did that mean the dimensions changed? What about the volume?

Phase 4 – drawing together

The pair who had recorded 6 x 2 x 2 were asked to explain what they meant by this and what each number represented, and what the answer meant. This led to discussing how to find the volume of a cuboid, with the introduction of the word 'formula'.

L x W X H was then presented on the board as the formula for calculating the volume of any cuboid.

As the lesson drew to a close, it was clear that some children were still unclear about precisely which numbers to multiply together, which would need to be returned to in a subsequent lesson.

Reflections on the lesson:

Pupils' engagement

The children worked well together in their mixed attainment pairs and were clear about what the task expected them to do.

Teaching approach

As the lead teachers walked around, they did provide some support by asking children to explain what they were doing and seeking clarification if an explanation was not clear.

The whole class discussion the different representations shared supported other children's understanding.

We felt we missed opportunities to discuss some ideas in greater depth, particularly what precisely is meant by length, width and height - do these change if a cuboid is rotated? This would have been a productive path to follow.

Setting up the problem

It might have been more productive to have initially started with every pair having the same number of cubes - the variation of different pairs focused on different total volumes got in the way of focusing on the key idea of the link between dimensions and volume.

Reflections on what we learned

Teaching

To take a risk and respond a little more openly to the responses of the children

To have the confidence to spend time on misconceptions that emerged.

Building in opportunities for discussion not only benefits the children's understanding, it also benefits the teacher's understanding and assessment of the children's learning.

Planning

Well-planned problems can lead both to consolidation and new learning.

Learning

Asking children to explore and investigate enables them to dive deeply into a concept

Learning through problem solving helps children develop a deeper concept of the big idea they are learning about.

Timing

Treating the time on the problem as open ended rather than determined by the end of the lesson is time well spent. Allow as much time as necessary.

Part 2: Drawing Together

At the end of the project, the teachers reflected on what issues and insights the research had raised regarding learning, teaching and professional development. The following sections summarise the teachers responses, each of which is followed by a commentary from Mike offering his further reflections.

Learning

Teachers reflections on what this research and style of teaching made them think about learning:

- Learning outcomes can vary, and different outcomes can be valuable.
- There is power in giving children the opportunity to play with a mathematical problem in a real world context.
- Children can be given the opportunity to discover, rather than be shown.
- There are benefits in children's collaboration that help develop mathematical talk.
- There is a need to consider when to have children take 'ownership' of their work and when to share anonymised versions of learner work.
- There is value in focusing on the processes of learning, including; learning from misconceptions, discussion of ideas, not only focussing on the answer, learning from each other, opportunities to reason, explain and question.
- The power of exploring many methods to a solution, rather than focusing on one solution method.
- There are benefits in providing children with opportunities to lead and explore.

Mike's Commentary

I find it helpful to see learning as a mix of sense making and meaning making. Sense making is the individual sense that a learner makes from the experiences they have, while meaning making is the collective mathematically agreed 'take' on experiences. For example, from working on multiplication with whole numbers, many children develop the sense that multiplication is an operation that always makes quantities larger, and consequently multiplying by a fraction does not make sense to them. Meaning making comes about through the collective sharing of the different senses that children are making of a situation. No matter how well we think we might have explained the meaning of something to the children, we cannot assume that the individual senses that they make of that explanation will always be the same as the intended meaning. I am reminded of a child who was adamant that 9 is an even number - when asked why, she showed how nine cubes could be made into three 'even' towers. The sense that she had made of the concept of even was that of being able to divide a number into any number of even (equal) groups rather than precisely two groups. As the reflections indicate, discussion can bring out the different 'senses' that children are making and help move everyone towards common meanings.

Issues teachers identified

- Children may find the experience stressful
- Inclusivity - are such lessons SEN friendly?
- Not all children enjoy learning this way.
- Limitations in children's ability to explain and discuss their own learning.

Mike's Commentary

How many of these issues are to do with schooling rather than learning? If children are used to a style of teaching mathematics which largely revolves around being shown what to do followed by practicing what they have been shown then they will doubtless find a shift to a problem-solving approach stressful. But in schools where such lessons have been the norm all the way through the years, then children come to accept that learning mathematics does mean thinking deeply and they develop, over time, the skills in explaining their thinking. Whether or not such lessons are any more or less able to engender inclusive than other mathematics lessons I think is open to further inquiry. If the problem selected is carefully chosen, well set up and individuals given appropriate support, then, in my experience, such lessons are highly inclusive, as, once everyone had found a solution to the problem in their own way, then everyone had something to contribute to the ensuing conversation.

Teaching

Teachers' reflections on what this research and style of teaching made them think about teaching included;

- The need to really think about the mathematical direction you want to move the children in.
- The opportunities a problem-solving approach provides for you to learn a lot about the children's thinking - a great assessment tool.
- How it enables teachers to unlock children's prior knowledge.
- The need to consider carefully exactly what the role of the teacher is, particularly in terms of choosing when or not to intervene and which of children's solutions might provide productive paths to follow-up.
- How it makes it possible to identify misconceptions.
- How it reveals children's attitudes towards their learning and understanding.
- That there does not need to be an 'end' answer for the work to be productive.
- That such lessons might be good for beginning and/or ending a unit of work.
- It enables a range of mathematics to be explored within one problem
- It extends children's reasoning and problem solving, naturally extending their learning.
- Teaching through problem solving is an inclusive way to teach
- The importance, when choosing and planning a problem of anticipating children's responses.

Mike's commentary

The teachers seemed split on whether or not they saw this as an inclusive way to teach - the group initially sharing reflection on teaching strongly thought it was, the group initially focusing on learning were less convinced. As noted, this is something that could be explored further. I, personally, try to avoid talking about children learning something 'naturally' or through 'discovery' - teaching through problem solving is, for me, is a highly directed form of teaching. It is just that the 'teaching' is largely being addressed at the planning stage through the cycle of selecting a problem, anticipating children's solution methods, adapting the problem if these methods seem unlikely to be productive in the classroom, anticipating solutions to the adapted problem and so on. A popular misconception to this way of teaching and learning is that it simply requires finding a problem that is likely interest the children and that addresses some mathematics - such an approach is highly unlikely to lead to a productive lesson. The importance of anticipating children's approaches lies in the control the teacher then has within the lesson when it comes to selecting which solution methods to share with everyone - these have to be chosen so that the core mathematics being learned can clearly and explicitly be drawn out, otherwise the discussion is reduced to simply being 'show and tell' (Stein et al., 2008).

Issues teachers identified

- Time
- Practicality of implementing in the 'everyday' classroom
- Strong teacher subject knowledge is required
- The need to carefully choose suitable and supportive resources
- The need to know your children

Mike's commentary

I would agree that planning a productive teaching through problem solving lesson requires strong subject knowledge. Planning in groups means that the collective subject knowledge is going to be greater than any one individual's. I am certainly not advocating that every, or even, many, lessons would be like this. A school starting to explore teaching through problem solving might start with just one such lesson every half term.

Professional Development

Teachers' reflections on what this research and style of teaching made them think about this as a form of professional development.

- The value of visiting others schools and observing others teach.
- The developing of collaborative networks and engaging in professional development'
- The opportunity for delving deeper into children's thinking'
- The learning coming about from sharing planning and having to justify choices.
- Developing the teacher's role as facilitator of learning

- Having time to examined the question ‘what does thinking look like?’ and coming to appreciate that there are different ways of thinking and engaging with mathematical concepts.
- Opportunities to try our problems in a safe supportive environment.
- Deepening understanding of variation in designing lessons.

Mike’s commentary

These reflections mirror much of what is noted in research more generally. There is a body of research showing the power of watching others teach, and that, although the person being observed learns a lot, it is the observers who learn the most (Joyce & Showers, 2002). There is also evidence of the power of teachers working together to think about what it is they want children to learn, with a focus on the sort of thinking that they want them to develop, rather than, as is more common, working together to plan teaching tasks (OECD, 2009). Getting clearer about desired learning outcomes and agreeing on these - that is, focusing on the attained curriculum - is more effective than only focusing on the intended curriculum.

Issues teachers identified

- Time constraints
- Pressure of being observed by others
- How to share with others.

Mike’s commentary

There is no doubt that this type of professional development is different to more traditional approaches, in that it explores one or two aspects of learning mathematics in great depth rather than trying to cover a breadth of ideas. The evidence of the sort of ‘mindset’ that this way of working engenders - in particular the importance of anticipating what sense children might make of the mathematics offered to them - is that, over time, it can shift teachers thinking (Takahashi & McDougal, 2016), but it has to be seen as a long-term goal, not a ‘one-shot’ experience. Ultimately, the best method of sharing, and getting used to being observed by others, is for schools to set up their own lesson studies.

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